PERFORMANCE ANALYSIS OF PAPR REDUCTION TECHNIQUES IN MULTICARRIER MODULATION SYSTEM

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is one of the many multicarrier modulation techniques which provide high spectral efficiency, less vulnerability to echoes, low implementation complexity and resilience to non-linear distortion. It is used in communication systems due to its various advantages. However, while this system is implemented problem of high peak to average power ratio (PAPR) is encountered. The reason behind this drawback is the existence of many independent subcarriers, due to which the signal amplitude can have high peak values as compared to average of whole system. The high PAPR in multicarrier transmission systems causes power degradation and spectrum spreading. Interleaving, Tone Reservation, Peak Reduction Carrier, Block Coding, Active Constellation Extension, Envelope Scaling among many PAPR reductions schemes that have been proposed as a remedy to this problem. In this paper, performances of Amplitude Clipping and Filtering, Selected Level Mapping (SLM), and Partial Transmit Sequence (PTS) techniques of PAPR reduction in OFDM systems by parameter variations are analyzed, based on Complementary Cumulative Distribution Function. An attempt has been made to simulate clipping and filtering technique with iterations and the simulation shows that PAPR problem is reduced as number of iterations increases. The attempts have also been made to simulate SLM technique and PTS technique by varying number of phase sequences, number of sub-blocks in SLM, PTS respectively and simulation results shows that by increasing the number of phase sequences, sub-blocks, PAPR can be reduced significantly. The mathematical equations are incorporated here to compute the maximum expected PAPR from an OFDM signal which shows when there is phase alignment of all subcarriers and sub carriers are equally modulated, then signal peak value hits the maximum. Besides these computer simulations, a comparative study of these three techniques is done.

Index Terms: Multi-Carrier Modulation (MCM), Orthogonal Frequency Division Multiplexing (OFDM), Peak-To-Average Power Ratio (PAPR), Complementary Cumulative Distribution Function (CCDF), Repeated Clipping and Filtering (RCF), Partial Transmit Sequence (PTS), Selected Level Mapping (SLM), Clipping Ratio (CR).

1. INTRODUCTION

Multi-Carrier Modulation (MCM) is a technique that has recently seen gaining popularity in wireless and wireline applications. In the past few years wireless communications have experienced a rapid growth due to the high mobility that they allow. However, wireless channels have some disadvantages, like signal fading due to multipath, that make them difficult to deal with. Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique that efficiently deals with selective fading channels. The advancing capabilities of digital signal processors make this technique of utmost interest.

Moreover for wireless applications, OFDM-based systems are of great interest since they provide a greater immunity to impulse noise and fast fading and eliminate the need for equalizers. The signal processing techniques like Fast Fourier transform (FFT) enables efficient hardware implementations for small numbers of carriers and make their realization simpler. OFDM (orthogonal frequency division multiplexing) has been proposed for many different types of systems from television broadcasting to wireless LANs (local area networks).

OFDM is based on the principle of splitting a high-rate data stream into a number of lower rate streams which are transmitted simultaneously on number of subcarriers. These subcarriers are overlapped with each other. As duration of symbol increases for lower rate parallel subcarriers, the relative amount of dispersion caused by multipath delay spreads decreases. The introduction of a guardtime in every OFDM symbol eliminates Inter-symbol interference (ISI) almost completely.

OFDM faces several challenges. The major challenge is the large peak-to-average ratio due to nonlinear behavior of amplifier. Large peak-to-average power ratio (PAPR) distorts the signal if the transmitter contains nonlinear components such as power amplifiers (PAs). This nonlinear
distortion causes both-in-band radiation and out-of-band interference to signals. Therefore for distortion less transmission, the power amplifiers require a back off which is approximately equal to the PAPR. This decreasesthe efficiency for amplifiers. This is the reason which arises the need for reducing the high PAPR.

PAPR can be analyzed by its complementary cumulative distribution function (CCDF). In this probabilistic approach certain methods have been proposed by researchers including constellation mapping, phase optimization, Tone Reservation (TR) and Tone Injection (TI) [9,10], coding schemes [8], nonlinear commanding transforms, Partial Transmission Sequence (PTS) and Selective Mapping (SLM) [4].

There are certain parameters like data rate loss, implementation complexity, capacity of PAPR reduction, transmission power, Bit-Error-Rate (BER) etc. and an effective PAPR reduction technique should be given the best trade-off between these parameters. However, simple PAPR reductions can be achieved by the proposed repeated clipping and filtering method in this paper. Further, this work presents PAPR reduction technique based on selective mapping (SLM) under different phase sequences V and PTS under different sub-blocks M.

The remainder of this paper is organized as follows. Section II presents some basics about PAPR problem in OFDM. Section III describes PAPR reduction techniques. In Section IV the overall analysis of these techniques amplitude clipping, SLM and PTS is given. Simulation results are shown in Section V. Section VI concludes the article.

2. PEAK-TO-AVERAGE POWER RATIO (PAPR)

The presence of large number of independently modulated sub-carriers in an OFDM system results in the high peak value of OFDM signal as compared to the average signal. This ratio of its peak to average power wavelenimate denoted as Peak-to-Average Power Ratio. Besides, the coherent addition of signals with same phase generates a peak which is N times the average signal.

2.1 PAPR of A Multicarrier Signal

Let the block of data of length N be represented by a vector \( X = [X_0, X_1, \ldots, X_{N-1}]^T \). The symbol duration of any symbol \( X_k \) in the set \( X \) is \( T \) and it represents one of the set of sub-carriers. The sub-carriers which are chosen to transmit the signal are orthogonal to each other. So it gives \( f_n = n\Delta f \) where \( n\Delta f = 1/NT \) and NT is the duration of the OFDM data block \( X \).

The complex block of data for the OFDM signal to be transmitted is –

\[
x(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n k} \text{ for } 0 \leq k \leq NT (1)
\]

The PAPR of the signal to be transmitted is defined as

\[
PAPR = \max_{0<k<NT} |x(k)|^2
\]

The main aim of PAPR reduction techniques is to reduce the \( \max|x(k)| \).

2.2 Effects of high PAPR

The number of sub carriers is very large in typical OFDM systems as a result of which the amplitude of the transmitted signal has a large dynamic range. It leads to in-band noise and out-of-band radiation when the signal is allowed to pass through the nonlinear region of PAs.

Although the problem mentioned above can be avoided by operating the amplifier in its linear region, but this results in a reduced power efficiency.

Besides, it also increases the complexity of analog to digital and digital to analog converters.

2.3 Complementary Cumulative Distribution Function (CCDF)

The Cumulative Distribution Function (CDF) is one of the most commonly used parameters to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF and it helps to measure the probability that the PAPR of a certain data block exceeds the given threshold.

The CDF of the amplitude of a signal sample is given by-

\[
F(z) = 1 - \exp(-x)
\]

The CCDF of the PAPR of block of data is desired to analyze the performance of various peak reduction techniques.

\[
P(P_{\text{PAPR}} > x) = 1 - P(P_{\text{PAPR}} \leq x) = 1 - (1 - \exp(f_z))\quad \text{(3)}
\]

2.4 Maximum expected PAPR from an OFDM signal

In an OFDM system, the high data rate information is grouped into smaller data which are placed orthogonal to each other. It is basically the sum of multiple sinusoids of having frequency separation 1/T where each sinusoid gets modulated by independent information \( b_n \). Mathematically,

Transmitted signal is-
\[ x(t) = \sum_{0}^{N-1} b_n e^{j2\pi nt/T} \]

Assuming, \( b_n = 1 \),
Therefore peak value of signal is-

\[ \max[x(t)x^*(t)] = \max \left[ \sum_{0}^{N-1} b_n e^{j2\pi nt/T} \right] \]

\[ = \max \left[ b_n b_n^* \sum_{0}^{N-1} e^{j2\pi nt/T} \right] \]

\[ = N^2 \]

And

Mean Square Value -

\[ E[x(t)x^*(t)] = E \left[ \sum_{0}^{N-1} b_n e^{j2\pi nt/T} \sum_{0}^{N-1} b_n^* e^{-j2\pi nt/T} \right] \]

\[ = E \left[ b_n b_n^* \sum_{0}^{N-1} e^{j2\pi nt/T} \right] \]

\[ = N \]

\[ \text{PAPR} = \frac{N^2}{N} \]

Therefore, \( \text{PAPR} = N \) \hspace{0.5cm} (4)

It is clear from (4), that for given N subcarriers and all sub-carriers are given same Modulation maximum expected PAPR from a OFDM signal is N.

3. PAPR REDUCTION TECHNIQUES

Several PAPR reduction techniques have been proposed in the literature[6]. The PAPR reduction techniques vary in accordance with the needs of system and are dependent on various factors such as BER increase, data rate loss, computation complexity, transmit power increase e.t.c.

These techniques are divided into two groups - signal distortion techniques and data scrambling techniques which are as follows-

3.1) Signal Distortion Techniques

3.1.1 Clipping and Filtering

3.1.2 Peak Reduction Carrier

3.2) Data Scrambling Techniques

In scrambling techniques, each OFDM signal is mixed with different scrambling sequences and the signal which has smallest PAPR value is transmitted.

3.2.1 Selected Mapping (SLM)

3.2.2 Partial Transmit Sequence (PTS)

3.2.3 Block Coding Techniques

3.2.4 Tone Reservation (TR)

3.1.1 Clipping and Filtering

Amplitude clipping is one of the simplest techniques for PAPR reduction in OFDM system. In this technique, initially a threshold value of amplitude is set and any sub-carrier having more amplitude compared to the threshold is clipped or that sub-carrier is filtered to lower PAPR value[4]. Basically, Clipping works on the idea of reducing large peaks by non-linearly distorting the signal. It does not scramble the signal and too large peaks occurs less often so the signal is seldom distorted. The maximum peak power allowed is decided by the system specifications, generally by the linear region of the power amplifier.

Mathematically,

\[ C(x) = \begin{cases} x, & |x| \leq k \\ k e^{b_p(x)}, & |x| > k \end{cases} \]

Where, \( C(x) \) is the amplitude value after clipping, \( k \) is the threshold set by user and \( x \) is the initial value of signal.

The Clipping Ratio (CR) can be determined by-

\[ CR = 20 \log_{10} \frac{r_x}{m} \text{db} \hspace{0.5cm} (6) \]

Where \( r_x \) is the rms value of \( x \).

Clipping is a non-linear process which introduces in-band noise, also called clipping noise, out of band noise and inter-carrier interference, as a result of which the system performance is degraded and spectrum efficiency is affected. However, filtering after clipping can reduce out of band noise but it cannot reduce in-band distortion. Clipping may cause some peak re-growth and the signal after amplitude clipping and filtering will exceed the clipping level at few points. The proposed repeated filtering and clipping method can be implemented to solve this problem. However, the desired amplitude level is only achieved after several iteration of this technique.

3.2.1 Selected Level Mapping (SLM)

In this a set of some different blocks of data representing the information similar to the original data blocks are selected. The
data blocks with low PAPR are then selected for transmission.[6]

Selective Mapping (SLM) is used for lowering the peak to average transmit power of multicarrier system with selected mapping. A complete set of member signals is generated representing the same information in selected mapping, and then the most favorable signal is selected with low PAPR and transmitted.

\[ X_m = [x_{m,0}, x_{m,1}, \ldots, x_{m,N-1}]^T \]

(m = 0,1, ..., M – 1) such that \( \sum_{m=0}^{M-1} x_m = X \) and the sub-blocks are combined to minimize the PAPR in the time domain. The S times over-sampled time domain signal of \( X_m \) is obtained by taking the IDFT of length \( N_s \) on \( X_m \) concatenated with (S-1)N zeros. Complex phase factors \( b_m = e^{j \theta_m} \), \( m = 0,1, \ldots, M – 1 \) are introduced to combine the PTS. The set of phase factors is denoted as vector \( b = [b_0, b_1, \ldots, b_{M-1}]^T \).

The time domain signal after combining is given by

\[ x'(b) = \sum_{m=0}^{M-1} b_m x_m(7) \]

Where, \( x'(b) = [x'_0(b), x'_1(b), \ldots, x'_{N-1}(b)]^T \)

PTS scrambles only part of sub-carriers. The basic principle behind this method is to divide original OFDM signal into many subsequences and they are further multiplied by different weights until an optimum value is selected.[4]

3.2.2 Partial Transmit Sequence (PTS):

In the PTS technique, input data block \( X \) is partitioned in \( M \) disjoint sub – blocks.

**Fig -1:** Block diagram of SLM technique

Each block of data is multiplied by \( V \) different phase sequences, having length \( N_b_v = [b_{v,0}, b_{v,1}, \ldots, b_{v,N-1}]^T \) resulting in \( V \) modified blocks. Thus, the \( V \) th phase sequence after multiplication is –

\[ X' = [X_0 b_{v,0}, X_1 b_{v,1}, \ldots, X_{N-1} b_{v,N-1}]^T \] (\( v = 0,1, \ldots, V – 1 \)).

Among the data blocks \( X'(v = 0,1, \ldots, V – 1) \), only the lowest PAPR data block is selected for transmission and the corresponding selected phase factors \( b_{v,n} \) should also be sent as side information to the receiver [6][7]. Amount of PAPR reduction for SLM depends on the number of phase sequences and the design of phase sequences. This technique applies scrambling rotation independently to all sub-carriers.

The positive side of selected mapping method is that it doesn’t eliminate the peaks, and can handle large number of sub-carriers.

The limitation of this method is the overhead of side information that requires to be sent to the receiver in order to reproduce information.

**Fig -2:** Block diagram of PTS technique

This method is flexible and efficient for OFDM system. The PTS method is a modified method of SLM. The merit of this method is that there is no need to transmit any side information to the receiver \( s \), when differential modulation is applied in all sub blocks.[4]

4. STUDY OF DIFFERENT PAPR REDUCTION TECHNIQUES

The PAPR reduction technique should be selected with awareness according to various system requirements.
### Table 1: Comparative Study of PAPR reduction Techniques

<table>
<thead>
<tr>
<th>Technique</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clipping and Filtering</td>
<td>No data rate loss, No transmit power increase</td>
<td>Signal Distortion</td>
</tr>
<tr>
<td>Selected Mapping(SLM)</td>
<td>Independent of number of carriers, Distortionless</td>
<td>Side information needed, data rate loss</td>
</tr>
<tr>
<td>Partial Transmit Sequence (PTS)</td>
<td>Less complex, Distortionless</td>
<td>Side information needed, data rate loss</td>
</tr>
</tbody>
</table>

There are many issues to be considered before using the PAPR reduction techniques in a digital communication system. These issues include PAPR reduction capacity, loss in data rate, transmit power increase, BER increase at the receiver, computational complexity increase and so on.

Table 1 shows that all PAPR reduction techniques have some advantages and disadvantages and are based on particular aspects of the system. These PAPR reduction techniques should be chosen carefully, forgetting the desirable minimum PAPR. For instance, if BER is considered to be crucial requirement of the system, SLM or PTS can be chosen. However, if transmit power, data rate is to be considered then Clipping and filtering is the best solution.

### 5. SIMULATION RESULTS

**5.1. Simulation 1**

Fig -3: PAPR’s CCDF using different number of sub-bands (N)

**5.2. Simulation 2**

Fig -4: PAPR’s CCDF using Repeated Clipping and Filtering (RCF) with different Clipping levels.

**5.3. Simulation 3**

Fig -5: SLM method with different Phase sequences

**5.4. Simulation 4**

Fig -6: PTS method with different sub-blocks (M)
5.5. Simulation 5

Fig.07 shows the three PAPR reduction techniques. It shows CCDF performances of Repeated Clipping and Filtering, SLM and PTS. The RCF is simulated with iterations I= [1,2,3,4] which shows 6.5 dB PAPR reduction with number of iterations equivalent to four as compared to 3.2 dB reduction when only one iteration is applied. Obviously, performance of PAPR reduction of clipping technique is improving when iterations are increased. However, at CCDF = 10^{-3}, simulation shows 6.5 dB, 3 dB, 1.6 dB PAPR reduction for RCF, SLM, PTS respectively compared to the original OFDM signal. Therefore, from fig.07, it can be observed that Repeated Clipping and Filtering method gives a better PAPR reduction performance than SLM and PTS.

CONCLUSIONS

OFDM is a very efficient technique for multicarrier transmission and for high-speed data transmission; it has become one of the standard choices. It has many advantages, but also has one major drawback - it has a very high PAPR. The simulation results show that clipping scheme can improve its performance of PAPR reduction by reducing its clipping levels. SLM performs better when the number of phase sequences is increased and performance of PTS is improved with increase in number of sub-blocks.

In contrast to conventional Amplitude Clipping method, Repeated Clipping and Filtering is proposed which gives better PAPR reduction performance than SLM and PTS.

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REFERENCES


BIOGRAPHIES:

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